

| | abc | $\neg abc$ | $a\neg bc$ | $ab\neg c$ | $\neg a\neg bc$ | $\neg ab\neg c$ | $a\neg b\neg c$ | $\neg a\neg b\neg c$ |
|--|-------|------------|------------|------------|-----------------|-----------------|-----------------|----------------------|
| φ | 0 | 0 | 0.5 | 0 | 1 | 0 | 0 | 0.5 |
| ψ | 0.5 | 1 | 0.5 | 0 | 0 | 0 | 0 | 0 |
| $\delta_{c(\psi)}^1(\varphi)$ | 0.5 | 0 | 0.5 | 0 | 1 | 0.5 | 0.5 | 1 |
| $\delta_{c(\psi)}^2(\varphi)$ | 0.5 | 0 | 0.5 | 0.5 | 1 | 1 | 1 | 1 |
| $\delta_{c(\psi)}^3(\varphi)$ | 0.5 | 0 | 0.5 | 1 | 1 | 1 | 1 | 1 |
| $\delta_{c(\psi)}^4(\varphi)$ | 0.5 | 0 | 0.5 | 1 | 1 | 1 | 1 | 1 |
| $d(0) = \delta_{c(\psi)}^0(\varphi)$ | 0 | 0 | 0.5 | 0 | 1 | 0 | 0 | 0.5 |
| $d(1) = \delta_{c(\psi)}^1(\varphi) \wedge c(\delta_{c(\psi)}^0(\varphi))$ | 0.5 | 0 | 0.5 | 0 | 0 | 0.5 | 0.5 | 0.5 |
| $d(2) = \delta_{c(\psi)}^2(\varphi) \wedge c(\delta_{c(\psi)}^1(\varphi))$ | 0.5 | 0 | 0.5 | 0.5 | 0 | 0.5 | 0.5 | 0 |
| $d(3) = \delta_{c(\psi)}^3(\varphi) \wedge c(\delta_{c(\psi)}^2(\varphi))$ | 0.5 | 0 | 0.5 | 0.5 | 0 | 0 | 0 | 0 |
| $d(4) = \delta_{c(\psi)}^4(\varphi) \wedge c(\delta_{c(\psi)}^3(\varphi))$ | 0.5 | 0 | 0.5 | 0 | 0 | 0 | 0 | 0 |

Table 1: Values of conditional dilations and fuzzy distances $d(n) = d(\omega, (\varphi, \psi))(n)$ for the example in Figure 7.

the comparison methods, and at exploring the suggested applications as well as new ones, including the choice of the most appropriate definition.

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